

Paper Reference 9MA0/02
Pearson Edexcel
Level 3 GCE

Mathematics
Advanced
Paper 2: Pure Mathematics 2

Wednesday 14 October 2020 – Afternoon

Time: 2 hours plus your additional time allowance.

MATERIALS REQUIRED FOR EXAMINATION
Mathematical Formulae and Statistical Tables (Green),
calculator

ITEMS INCLUDED WITH QUESTION PAPER
Diagram Book
Answer Book

Q66786A

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

INSTRUCTIONS

In the boxes on the Answer Book and on the Diagram Book, write your name, centre number and candidate number.

Answer ALL questions and ensure that your answers to parts of questions are clearly labelled.

Answer the questions in the Answer Book or on the separate diagrams – there may be more space than you need.

Do NOT write on the Question Paper.

You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

Inexact answers should be given to three significant figures unless otherwise stated.

INFORMATION

A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.

**There are 16 questions in this Question Paper.
The total mark for this paper is 100**

**The marks for each question are shown in brackets
– use this as a guide as to how much time to spend on
each question.**

ADVICE

**Read each question carefully before you start to answer
it.**

Try to answer every question.

Check your answers if you have time at the end.

1. Refer to the table for Question 1 in the Diagram Book.

It shows corresponding values of x and y for

$$y = \sqrt{\frac{x}{1+x}}$$

The values of y are given to 4 significant figures.

- (a) Use the trapezium rule, with all the values of y in the table, to find an estimate for

$$\int_{0.5}^{2.5} \sqrt{\frac{x}{1+x}} \, dx$$

giving your answer to 3 significant figures.

(3 marks)

(continued on the next page)

1. continued.

(b) Using your answer to part (a), deduce an estimate for

$$\int_{0.5}^{2.5} \sqrt{\frac{9x}{1+x}} \, dx$$

(1 mark)

Given that

$$\int_{0.5}^{2.5} \sqrt{\frac{9x}{1+x}} \, dx = 4.535$$

to 4 significant figures

(c) comment on the accuracy of your answer to part (b)

(1 mark)

(Total for Question 1 is 5 marks)

2. Relative to a fixed origin, points **P**, **Q** and **R** have position vectors \underline{p} , \underline{q} and \underline{r} respectively.

Given that

- **P**, **Q** and **R** lie on a straight line
- **Q** lies one third of the way from **P** to **R**

show that

$$\underline{q} = \frac{1}{3} (\underline{r} + 2\underline{p})$$

(Total for Question 2 is 3 marks)

3. (a) Given that

$$2 \log (4 - x) = \log (x + 8)$$

show that

$$x^2 - 9x + 8 = 0$$

(3 marks)

(b) (i) Write down the roots of the equation

$$x^2 - 9x + 8 = 0$$

(ii) State which of the roots in (b)(i) is NOT a solution of

$$2 \log (4 - x) = \log (x + 8)$$

giving a reason for your answer.

(2 marks)

(Total for Question 3 is 5 marks)

Turn over

4. In the binomial expansion of

$(a + 2x)^7$ where a is a constant

the coefficient of x^4 is 15 120

Find the value of a

(Total for Question 4 is 3 marks)

5. The curve with equation $y = 3 \times 2^x$ meets the curve with equation $y = 15 - 2^{x+1}$ at the point P

Find, using algebra, the exact x coordinate of P

(Total for Question 5 is 4 marks)

6. (a) Given that

$$\frac{x^2 + 8x - 3}{x + 2} \equiv Ax + B + \frac{C}{x + 2}$$

$$x \in \mathbb{R} \quad x \neq -2$$

find the values of the constants **A**, **B** and **C**
(3 marks)

(b) Hence, using algebraic integration, find the exact value of

$$\int_0^6 \frac{x^2 + 8x - 3}{x + 2} dx$$

giving your answer in the form **a + b ln 2**
where **a** and **b** are integers to be found.
(4 marks)

(Total for Question 6 is 7 marks)

7. Refer to the diagram for Question 7 in the Diagram Book.

It shows a sketch of the curve **C** with equation

$$y = \frac{4x^2 + x}{2\sqrt{x}} - 4\ln x \quad x > 0$$

- (a) Show that

$$\frac{dy}{dx} = \frac{12x^2 + x - 16\sqrt{x}}{4x\sqrt{x}}$$

(4 marks)

The point **P**, shown in the diagram, is the minimum turning point on **C**

- (b) Show that the **x** coordinate of **P** is a solution of

$$x = \left(\frac{4}{3} - \frac{\sqrt{x}}{12} \right)^{\frac{2}{3}}$$

(3 marks)

(continued on the next page)

Turn over

7. continued.

(c) Use the iteration formula

$$x_{n+1} = \left(\frac{4}{3} - \frac{\sqrt{x_n}}{12} \right)^{\frac{2}{3}} \quad \text{with } x_1 = 2$$

to find

(i) the value of x_2 to 5 decimal places,

(ii) the x coordinate of P to 5 decimal places.

(3 marks)

(Total for Question 7 is 10 marks)

8. A curve **C** has equation $y = f(x)$

Given that

- $f'(x) = 6x^2 + ax - 23$ where **a** is a constant
- the **y** intercept of **C** is -12
- $(x + 4)$ is a factor of $f(x)$

find, in simplest form, $f(x)$

(Total for Question 8 is 6 marks)

9. A quantity of ethanol was heated until it reached boiling point.

The temperature of the ethanol, $\theta^{\circ}\text{C}$, at time t seconds after heating began, is modelled by the equation

$$\theta = A - Be^{-0.07t}$$

where A and B are positive constants.

Given that

- the initial temperature of the ethanol was 18°C
- after 10 seconds the temperature of the ethanol was 44°C

- (a) find a complete equation for the model, giving the values of A and B to 3 significant figures.
(4 marks)

(continued on the next page)

Turn over

9. continued.

Ethanol has a boiling point of approximately 78°C

(b) Use this information to evaluate the model.
(2 marks)

(Total for Question 9 is 6 marks)

10. In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\cos 3A \equiv 4 \cos^3 A - 3 \cos A$$

(4 marks)

(b) Hence solve, for $-90^\circ \leq x \leq 180^\circ$, the equation

$$1 - \cos 3x = \sin^2 x$$

(4 marks)

(Total for Question 10 is 8 marks)

Turn over

11. Refer to the diagram for Question 11 in the Diagram Book.

It shows a sketch of the graph with equation

$$y = 2|x + 4| - 5$$

The vertex of the graph is at the point **P, shown in the diagram.**

(a) Find the coordinates of **P**
(2 marks)

(b) Solve the equation

$$3x + 40 = 2|x + 4| - 5$$

(2 marks)

(continued on the next page)

11. continued.

A line **L** has equation $y = ax$, where **a** is a constant.

Given that **L** intersects $y = 2|x + 4| - 5$ at least once,

- (c) find the range of possible values of **a**, writing your answer in set notation.
(3 marks)

(Total for Question 11 is 7 marks)

12. Refer to the diagram for Question 12(a) in the Diagram Book.

The curve shown in the diagram has parametric equations

$$\mathbf{x = 6 \sin t \qquad y = 5 \sin 2t \qquad 0 \leq t \leq \frac{\pi}{2}}$$

The region R , shown shaded in the diagram, is bounded by the curve and the X -axis.

(a) (i) Show that the area of R is given by

$$\int_0^{\frac{\pi}{2}} 60 \sin t \cos^2 t \, dt$$

(3 marks)

(ii) Hence show, by algebraic integration, that the area of R is exactly 20

(3 marks)

(continued on the next page)

Turn over

12. continued.

Refer to the diagram for Question 12(b) in the Diagram Book.

Part of the curve is used to model the profile of a small dam, shown **OMNP in the diagram.**

Using the model and given that

- **x and y are in metres**
- **the vertical wall of the dam is 4.2 metres high**
- **there is a horizontal walkway of width **MN** along the top of the dam**

(b) calculate the width of the walkway.
(5 marks)

(Total for Question 12 is 11 marks)

13. The function g is defined by

$$g(x) = \frac{3\ln(x) - 7}{\ln(x) - 2} \quad x > 0 \quad x \neq k$$

where k is a constant.

(a) Deduce the value of k
(1 mark)

(b) Prove that

$$g'(x) > 0$$

for all values of x in the domain of g
(3 marks)

(c) Find the range of values of a for which

$$g(a) > 0$$

(2 marks)

(Total for Question 13 is 6 marks)

14. A circle **C** with radius **r**

- lies only in the 1st quadrant
- touches the **X**–axis and touches the **y**–axis

The line **L** has equation $2x + y = 12$

- (a) Show that the **X** coordinates of the points of intersection of **L** with **C** satisfy

$$5x^2 + (2r - 48)x + (r^2 - 24r + 144) = 0$$

(3 marks)

Given also that **L** is a tangent to **C**,

- (b) find the two possible values of **r**, giving your answers as fully simplified surds.

(4 marks)

(Total for Question 14 is 7 marks)

15. In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

A geometric series has common ratio r and first term a

Given $r \neq 1$ and $a \neq 0$

(a) prove that

$$S_n = \frac{a(1-r^n)}{1-r}$$

(4 marks)

Given also that S_{10} is four times S_5

(b) find the exact value of r

(4 marks)

(Total for Question 15 is 8 marks)

Turn over

16. Use algebra to prove that the square of any natural number is EITHER a multiple of 3 OR one more than a multiple of 3

(Total for Question 16 is 4 marks)

TOTAL FOR PAPER IS 100 MARKS

END OF PAPER
